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The combinatorics of expansions in non-integer bases

Let $1 < \beta < 2$. We say that $x \in [0,1]$ *has a β-expansion* if

$$x = \sum_{n=1}^{\infty} \varepsilon_n \beta^{-n}, \quad \varepsilon_n \in \{0,1\}.$$

In my talk I will prove that Lebesgue almost every x has a continuum of β-expansions and will describe various ways of measuring such a continuum. In particular, if β is a Pisot number, then such a continuum has the same dimension (growth rate) for almost every x.

In the opposite direction, I will show that if β is sufficiently close to 2, then there are x with a unique β-expansion – and the closer β to 2, the more such x exist. When β is close to 2, then there are also x with a prescribed number of β-expansions (finite or infinite countable).

Time permitting, I will also talk about the cases when $\beta > 2$ and is a non-integer (with digits $0, 1, \ldots, m$ with some $m \geq \lfloor \beta \rfloor$), where similar results hold.

This talk is based on recent developments made by Komornik, Loreti, Glendinning, Feng, Baker, myself and many others.