COMBINATORIAL KY FAN'S LEMMA AND APPLICATIONS

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The combinatorial Ky Fan lemma is a powerful tool in combinatorial topology. In this talk, I will explain how to prove this result via a combinatorial Stokes formula and show some applications in topology (briefly) and in combinatorics.

The first combinatorial application I plan to present is the study of colorings of Kneser graphs. Kneser graphs form an important family of graphs, with many interesting properties, and which are defined as follows. The vertex set of a Kneser graph of parameters k and n is the collection of all k-subsets of a ground set of cardinality n. Two vertices form an edge if the corresponding k-subsets are disjoint. The chromatic number of these graphs had been a central conjecture in combinatorics before Lovász found a proof based on the Borsuk-Ulam theorem (1979). The combinatorial Ky Fan lemma can be used to provide an alternate combinatorial proof and further results on their colorings. The idea at the origin of this use is due to Matoušek (2004).

A second application I plan to present is a combinatorial proof of the splitting necklace theorem, due to Pálvölgyi (2009). This theorem states that if two thieves steal a necklace with t types of beads, there can fairly split the necklace with no more than t cuts, regardless of the total number of beads. The necklace is suppose to be open, with the beads strongly fixed on the string, and fairly mean that each thief receives almost the same amount of beads of each type.

Many questions remain open regarding the Ky Fan lemma and its applications. They will be emphasized during the talk.