La Tour d'Hanoï: . . . to 4 and beyond*

Andreas M. Hinz
LMU Munich (Germany)
hinz@math.lmu.de

Abstract. When it comes to the question of finding shortest paths between any two states of the Tower of Hanoi we need to model the game by its state graph, called *Hanoi graph* H_3^n, where 3 stands for the number of pegs (or diamond needles) and n for the number of (golden) discs. There is an *isomorphic* representation of these graphs, the *Sierpiński graphs* S_3^n, whose labelling is better suited for determining distances, as the automata of Romik have shown. Sierpiński graphs have the further advantage to be easily generalizable to an arbitrary number p of pegs, and Romik's algorithm finds an elegant extension to these graphs. Tower of Hanoi versions with more than 3 pegs had been proposed by Henry Ernest Dudeney, who called the $p = 4$ case *The Reve's Puzzle* at the beginning of the last century. But, alas!, the corresponding Hanoi graphs H_p^n are not isomorphic to S_p^n anymore, so that even the classical question to find a shortest solution to transfer a complete tower from one peg to another one remained an open problem for more than a century. This led to the *Frame-Stewart Conjecture* about the optimality of a certain algorithm which seems to have come to a decision, very recently and very close to the site of the summer school, at least for the case of The Reve's Puzzle.